and also to the small variations in the clock rate. Rarely does this difference reach 0.^{s} 015. Only during the spring are there greater variations, which are doubtless due to the variability of weather conditions. Perhaps the temperature has not been kept absolutely constant, or the pressure has not been adjusted every time that the Cp showed the necessity for it. We may conclude that today, with two good instruments available, in charge of skillful astronomers, and well-installed, under a sky which permits the determination of the time, on the average, every 2 or 3 days, the value of Cp may be found with a mean error of

$$\frac{\pm 0.^s01}{\sqrt{2}} = 0.^s\,0071$$

With one good clock checked against another we can forecast Cp for 3 or 4 successive days with an uncertainty amounting hardly to $\pm 0.^s\,02$.

It is a splendid triumph for astronomy!

GEOPHYSICS—*Geodetic constants.*[1] WALTER D. LAMBERT, U. S. Coast and Geodetic Survey.

The Newtonian constant of gravitation[2] and the mean density of the earth are so closely related that if one is known the other may be at once derived. The Newtonian constant is the quantity actually determined in the laboratory. The product of the two quantities is known within about one part in one hundred thousand, although neither quantity by itself is known within one part in ten thousand. The formula for the product may be written:

$$k\rho = \frac{3}{4\pi a}\left(g_\epsilon + \frac{3}{2}\,\omega^2\,a + \frac{3}{7}\omega^2\,a\,f\right)$$

Here k = Newtonian gravitation constant.

ρ = mean density of the earth.

a = equatorial radius of the earth considered as an ellipsoid of revolution.

g_ϵ = equatorial surface gravity.

ω = angular velocity of the earth's rotation, so that $\omega^2 a$ represents the centrifugal force of rotation at the Equator.

f = flattening (ellipticity) of the earth.

[1] Presented at the 977th Meeting of the Philosophical Society of Washington, October 13, 1928. The general subject of the papers given at the meeting was *Constants of Nature*. Received November 15, 1928.

[2] This paper followed one by Paul R. Heyl, in which the Newtonian constant of gravitation was discussed.

If we assume for the value of k that given by the preceding speaker, namely 6.664×10^{-8} c.g.s. units, and for the values of a and g_e those soon to be stated, the value of the mean density comes out as 5.522. This mean density may be considered as one of the geodetic constants to be discussed in this paper.

The dimensions of the earth and the coefficients in the formula for gravity at its surface are the principal remaining geodetic constants that we are to discuss. Let us consider the former first. Theoretically at least, it is not essential to have astronomical observations in order to determine the size and shape of the earth's physical surface, or any portion of it. The work could be done even though the heavens were perpetually covered with impenetrable clouds. Consider any portion of the earth's surface with a number of points on it, each point visible to its nearer neighbors. These points could be considered as the vertices of an irregular polyhedron. The distance between two vertices could be measured directly and would serve as a base line for what might be termed three-dimensional triangulation. The face angles of each polyhedral angle could then be measured, the plane of the divided circle used being made coincident with the plane of the face angle. From these data the size and shape of the polyhedron could be deduced. All this work would be quite independent of considerations of potential or of level surfaces or of latitude and longitude.

Practically, however, the accuracy of the results would be vitiated by atmospheric refraction, especially refraction in a vertical plane for objects near the horizon. This refraction, as every geodetic observer knows, is exceedingly irregular and tricky. The method just outlined has therefore only a theoretical interest. Actual determinations of the figure of the earth depend on astronomical observations, that is, determinations of latitude, longitude and azimuth over a given region, combined with large-scale surveying operations over the same region. The ancient Greeks must have done something of the sort, though their astronomical observations were of the roughest and their determinations of distance probably mere estimates based on travelers' accounts. Even so, however, the Greeks of the time of Aristotle[3]

[3] "Moreover those mathematicians who try to compute the circumference of the earth say that it is 400,000 stadia, which indicates not only that the earth's mass is spherical in shape but also that it is of no great size as compared with the heavenly bodies." Aristotle, *De Caelo*, Book II, Chap. 14. This passage follows a long argument in favor of the sphericity of the earth. Some of the arguments sound modern enough, others seem strange to our present ways of thinking. This seems to represent the first scientific attempt or attempts now on record to determine the size of the earth. No further details are given. The entire treatise has been translated by J. L. Stocks and published by the Clarendon Press, Oxford, in 1922.

had an approximate knowledge of the size of the earth considered as a sphere. About a century later Eratosthenes[4] obtained an even better approximation by a process identical in principle with that used by every geodesist down to the time when the electric telegraph became available for determining longitudes. Just how approximate these early determinations were we can not say because of the uncertainty regarding the exact modern equivalents of the linear units used.

Let us skip over some two thousand years and consider now the most modern determinations of the dimensions of the earth. As the most acceptable figures let us take those adopted in 1924 by the Section of Geodesy of the International Geodetic and Geophysical Union and defined by the parameters of the International Ellipsoid of Reference. These figures are based on Hayford's[5] discussion of geodetic operations in the United States only, but have been substantially confirmed by Heiskanen's[6] discussion of European triangulation and by other geodetic and astronomical evidence.[7]

The fundamental parameters are:

a (semi-major axis) = 6,378,388 meters

f (flattening or ellipticity) = 1/297.0

From these there result:

b (semi-minor axis) = 6,356,912 meters

Q (quadrant of a meridian) = 10,002,288 meters

It is seen that the meridian quadrant is over 2 kilometers longer than

[4] Eratosthenes, librarian at Alexandria, died about 195 B. C. We owe our knowledge of his geodetic work to a book by Cleomedes, a Greek writer who is supposed to have lived about 100 A. D. The account of Eratosthenes' work is in Chap. 10 of his book, the Latin title of which is *De Motu Circulari Corporum Celestium*. Eratosthenes' result is a circumference of 250,000 stadia. There is no certainty that the stadium of Aristotle and that of Eratosthenes represented the same length. If we use 185 meters, which is usually given as the length of the Attic stadium, we get for the circumference according to Eratosthenes some 46,000 kilometers, instead of the actual 40,000.

I am indebted to Mr. Otis Hill of the Coast and Geodetic Survey for invaluable help in connection with these and other references to classical literature.

[5] J. F. HAYFORD. *Supplementary investigation in 1909 of the fijure of the earth and isostasy.* Published by the U. S. Coast and Geodetic Survey, 1910.

[6] W. HEISKANEN. *Die Erddimensionen nach den europäischen Gradmessungen.* Veröff. Finn. Geod. Inst. 6. 1926. A slight revision of the conclusions from the same data is given by Heiskanen in the *Vierteljahrsschr. Astron. Ges.* **61** (Jahrgang 1926): 215.

[7] For references, see W. D. LAMBERT. *The figure of the earth and the new international ellipsoid of reference.* Science **63**: 242. 1926. A version revised by the author and translated into French by Col. Perrier appeared in the Bull. Géod. **10**: 81. 1926.

an even ten thousand kilometers, which was the figure aimed at when the metric system was devised.

What may be the probable limits of error of these figures (not the "probable errors" in the technical sense), it is rather difficult to say. Perhaps fifty meters in the semi-axes and a few tenths of a unit in the reciprocal of the flattening.

Let us make our formula for gravity at the surface of the earth consistent with the International Ellipsoid of Reference. We must then write as the value of gravity in cm/sec²:

$$g = \underset{\pm 8}{978.052}\,[1 + \underset{\pm 5}{0.005288}\sin^2\phi - 0.000006\sin^2 2\phi]$$

where ϕ = geographic latitude. Only the coefficients within the square brackets depend on the ellipticity. The coefficient outside, 978.052 cm/sec², is essentially independent of the dimensions of the earth and must be determined by observation. The value written down is the largest of all the more recent determinations[8] and this for two reasons: (1) There is reason to believe, as Bowie[9] has pointed out, that a more accurate reduction for the elevation of the station would slightly increase the values of gravity on land, and it is on these land values that our gravity formulas have hitherto been based; (2) The formula is meant to represent average conditions over the earth's entire surface, nearly three-fourths of which is ocean, and it appears from determinations of gravity at sea, which we are just beginning to obtain, that gravity at sea tends to be in excess of gravity on land even after the latter has been reduced for elevation. Bowie's suggested improvement in the method of reduction applies to sea stations also and should tend to harmonize the results for gravity stations on sea and on land.

I have written beneath the coefficients, estimates of their probable limits of error. They are largely matters of opinion, for a real basis of evaluation is lacking. The ±8 attached to the 978.052 is intended to include the error in the absolute determination of gravity, an exceedingly delicate and difficult operation when an accuracy of a few

[8] F. R. Helmert. *Neue Formeln für den Verlauf der Schwerkraft im Meeresniveau beim Festlande.* Sitzungsber. K. Preuss. Akad. Wiss. **1915**: 676.

W. Heiskanen. *Untersuchungen über Schwerkraft und Isostasie.* Veröff. Finn. Geod. Inst. 4.

[9] W. Bowie. *The effect of the shape of the geoid on values of gravity at sea.* Am. Journ. Sci. **14**: 222. 1927.

Rapport de la Sous-Commission spécialement chargée de déterminer les réductions à faire subir aux intensités observées en mer. Bull. Géod. **17**: 29. 1928.

parts in a million is sought. The authors of the most recent absolute determination, Kühnen and Furtwängler,[10] estimate the mean error of their result as ± 0.003 cm/sec^2.

To return to the apparent systematic difference between gravity on land and gravity at sea, this may be represented very roughly indeed by putting within the square brackets a longitude term such as:

$\pm 0.000023 \cos^2 \phi \cos 2 (\lambda + 5°)$, where λ = east longitude,

so that our formula becomes:[11]

$$g = 978.052 \left[1 + 0.005288 \sin^2 \phi - 0.000006 \sin^2 2 \phi + 0.000023 \cos^2 \phi \cos 2 (\lambda + 5°)\right]$$

The form of the added term is that of a surface spherical harmonic of the second degree and is conceivably the first of a long series of spherical harmonic terms related perhaps to the configuration of the lithosphere.[12] Parenthetically it should be said that other harmonic terms of the second degree and terms of lower degree are either already implicitly contained in the gravity formula or are omitted from it for sound theoretical reasons.

The presence of such a term is rather puzzling, for it implies an ability of the earth's crust to sustain the stresses due to a wide-spread and rather large excess or deficiency of matter, an ability not in accord with much other evidence. Yet, unless we are the victims of an uncommonly perverse combination of accidental errors, we can hardly escape attributing some reality to this longitude term. It does not rest solely on the recently discovered systematic difference between gravity on land and gravity at sea, some of which difference can be explained by Bowie's suggested improvement in methods of reducing for elevation. It appeared thirteen years ago in Helmert's[13] discussion of gravity observations all made on land. The longitude term in the gravity formula implies a corresponding term in the figure of the earth, making the geoid an ellipsoid of three unequal axes instead of an ellip-

[10] Fr. Kühnen and Ph. Furtwängler. *Bestimmung der absoluten Grösse der Schwerkraft zu Potsdam mit Reversionspendeln.* Veröff. K. Preuss. Geod. Inst. **27**. 1906.

[11] W. Heiskanen. *Ist die Erde ein dreiachsiges Ellipsoid?* Gerlands Beitr. Geophysik **19**: 356. 1928. Or in condensed form in the Astron. Nachr. **232** (5562): 305. 1928. The difference of one unit in the sixth decimal in the coefficients of $\sin^2\phi$ and $\sin^2 2\phi$ between the formulas of Heiskanen and of this article for the same flattening, 1/297, is due to the fact that Heiskanen's spheroid is not an exact ellipsoid.

[12] A. Prey. *Darstellung der Höhen- und Tiefenverhältnisse der Erde durch eine Entwickelung nach Kugelfunktionen bis zur 16. Ordnung.* Abh. K. Ges. Wiss. Göttingen. Math.-phys. Kl. **11**: 1. 1922.

[13] See note 8.

soid of rotation. Fifty years ago Clarke[14] deduced from triangulation and from astronomic determinations a figure of the earth that strongly suggests recent determinations of the longitude term in the gravity formula; furthermore, Heiskanen's recent discussion of European triangulation, in which discussion he allowed for the effects of topography and isostatic compensation, likewise tends to the same conclusion. So do results from discussions of the variation of latitude[15] and of the lunar parallax,[16] though these latter give at present only rough qualitative indications. All these results point to a difference between the maximum and minimum equatorial semi-axes of the order of two or three hundred meters, with the longer axis approximately in the plane of the meridian of Greenwich.

We should like to get a better hold on the real size of the longitude term and likewise to know whether it stands in the main by itself or whether it is only one of many spherical harmonic terms of about the same order of magnitude. If the latter, we should expect these terms to be related to the configuration of the continents and oceans. But if not, if this one longitude term stands practically alone, then perhaps we may see in it a vestige of some state of the earth as it was in the remote past when for some reason the earth was nearly a triaxial ellipsoid with one axis of the equator decidedly longer than the other. Perhaps we may imagine that this happened when the moon parted company with the earth, as in Darwin's theory, being expelled by the resonance effect of the solar tides at a time when the earth rotated much more rapidly than now. But this is frankly wild speculation and perhaps it will be well to close before we get too far away from observed facts. At any rate the longitude term, its reality, its size if real, and its geophysical significance, present one of the most interesting problems in present-day geodesy.

[14] A. R. Clarke. *On the fjure of the earth.* Lond. Edinb. Dubl. Philos. Mag. Journ. Sci. **6**: 81. 1878.

[15] W. D. Lambert. *An investigation of the latitude of Ukiah, Calif., and of the motion of the pole.* Coast & Geod. Surv. Spec. Pub. **80**: 59. 1922.

[16] W. D. Lambert. *The fjure of the earth and the parallax of the moon.* Astron. Journ. **38** (908): 181. 1928.